Main points:

1. Main principle of Photometric Stereo method

2. Reconstruction of surface colour and shape using PS method in presence of highlights and shadows

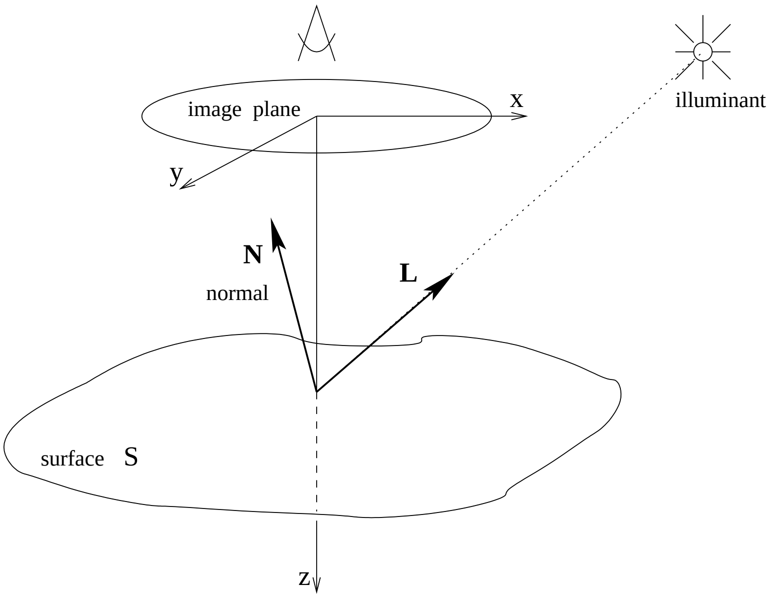
   - Separation of shape and colour descriptors
   - Effects of highlights and shadows. Coleman-Jain solution and its shortfalls
   - Colour differencing method and Normalised shadow thresholding
   - Unified algorithm and Results

3. Use of PS method for description and recognition of 3D textures

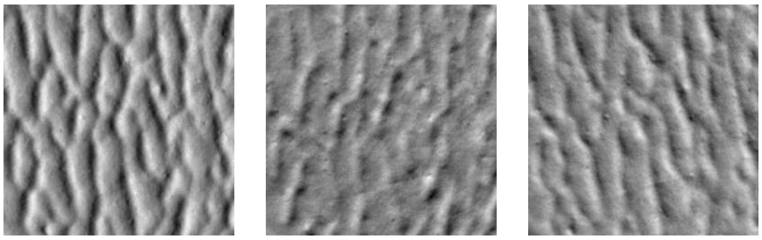
   - 3D textures. Texture features: variance of filtered output
   - Texture features and illumination. Surface normals and surface correlation matrices.
   - Response of texture features to illumination: compare two models
   - Recovery of illumination from a single image
   - Recognition of a 3D texture from a single image

4. Conclusion

Photometric Stereo for Lambertian surfaces



$$I = \rho \mathbf{L}^T \mathbf{n}$$



$$\mathbf{I} = \rho [L] \mathbf{n} \Rightarrow$$

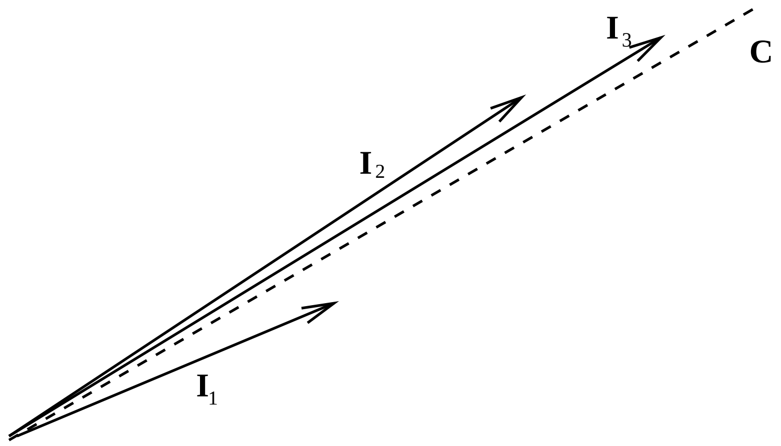
$$\rho \mathbf{n} = [L]^{-1} \mathbf{I}$$

Colour Lambertian Surfaces

Intensity scalar $\rightarrow$ colour vector

$$\mathbf{I}^k = \rho\left(\mathbf{L}^{k^T}\mathbf{N}\right)\mathbf{C} = s^k\mathbf{C}$$

$$[I] = \begin{bmatrix} I_1^1 & I_2^1 & I_3^1 \\ I_1^2 & I_2^2 & I_3^2 \\ I_1^3 & I_2^3 & I_3^3 \end{bmatrix} = \begin{bmatrix} s^1\mathcal{C}_r & s^1\mathcal{C}_g & s^1\mathcal{C}_b \\ s^2\mathcal{C}_r & s^2\mathcal{C}_g & s^2\mathcal{C}_b \\ s^3\mathcal{C}_r & s^3\mathcal{C}_g & s^3\mathcal{C}_b \end{bmatrix} = \mathbf{S}\mathbf{C}^T$$



Colour $\mathbf{C}$ is the principal eigenvector of $[I]^T[I]$

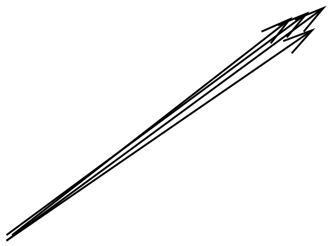
Shading $\mathbf{S}$ is the principal eigenvector of $[I][I]^T$

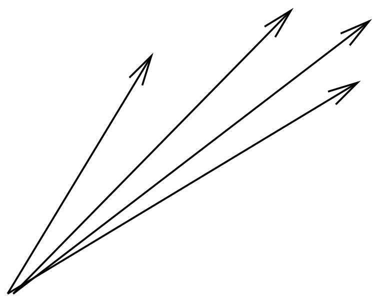
Highlights and shadows: problem

Highlights and shadows disturb the linearity. The colour and the normals become distorted.

Solution in absence of shadows (Coleman, Jain):

$$
\begin{array}{lll}
\mathbf{I}_1 & \mathbf{I}_1\mathbf{I}_2\mathbf{I}_3 \longrightarrow & \rho_1\mathbf{N}_1 \\
\mathbf{I}_2 & \mathbf{I}_2\mathbf{I}_3\mathbf{I}_4 \longrightarrow & \rho_2\mathbf{N}_2 \\
\mathbf{I}_3 & \mathbf{I}_3\mathbf{I}_4\mathbf{I}_1 \longrightarrow & \rho_3\mathbf{N}_3 \\
\mathbf{I}_4 & \mathbf{I}_4\mathbf{I}_1\mathbf{I}_2 \longrightarrow & \rho_4\mathbf{N}_4
\end{array}
$$





Disadvantage: does not differentiate between highlights and shadows

Problem detection: find where linearity is disturbed:

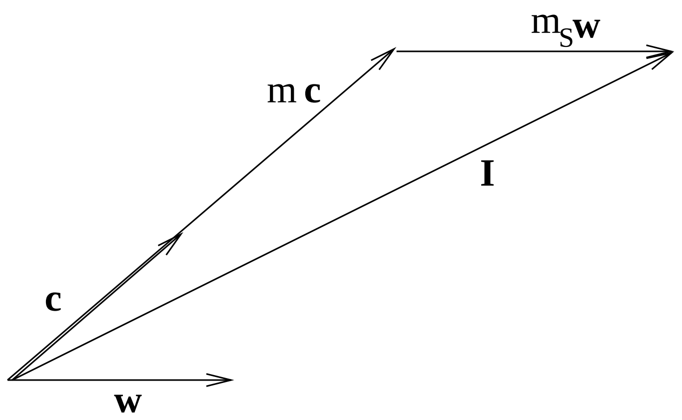
$$a_1\mathbf{L}^1 + a_2\mathbf{L}^2 + a_3\mathbf{L}^3 + a_4\mathbf{L}^4 = 0$$

$$a_1\rho(\mathbf{L}^1 \cdot \mathbf{n}) + a_2\rho(\mathbf{L}^2 \cdot \mathbf{n}) + a_3\rho(\mathbf{L}^3 \cdot \mathbf{n}) + a_4\rho(\mathbf{L}^4 \cdot \mathbf{n}) = 0$$

$$a_1 I^1 + a_2 I^2 + a_3 I^3 + a_4 I^4 = 0$$

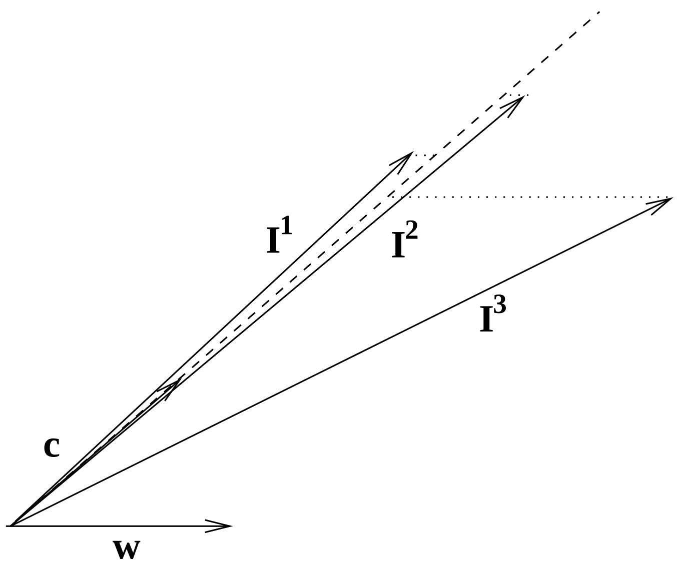
$$\Rightarrow \mathbf{a} \cdot \mathbf{I} = 0$$

Colour differencing method



Dichromatic reflection theory:

$$\mathbf{I} = \mathbf{C} + \mathbf{C}_S = m\mathbf{c} + m_S\mathbf{w}$$



It all goes wrong when **c** gets closer to **w**.

Normalised shadow thresholding

1. Determine whether the quadruple is problematic

2. Exclude the brightest pixel from the quadruple. The triplet:

   - does not contain a highlight (produced by $\mathbf{L}^b$)
   - can contain a shadow (produced by $\mathbf{L}^d$)

3. Recover the normal $\mathbf{n}$ from the triplet.

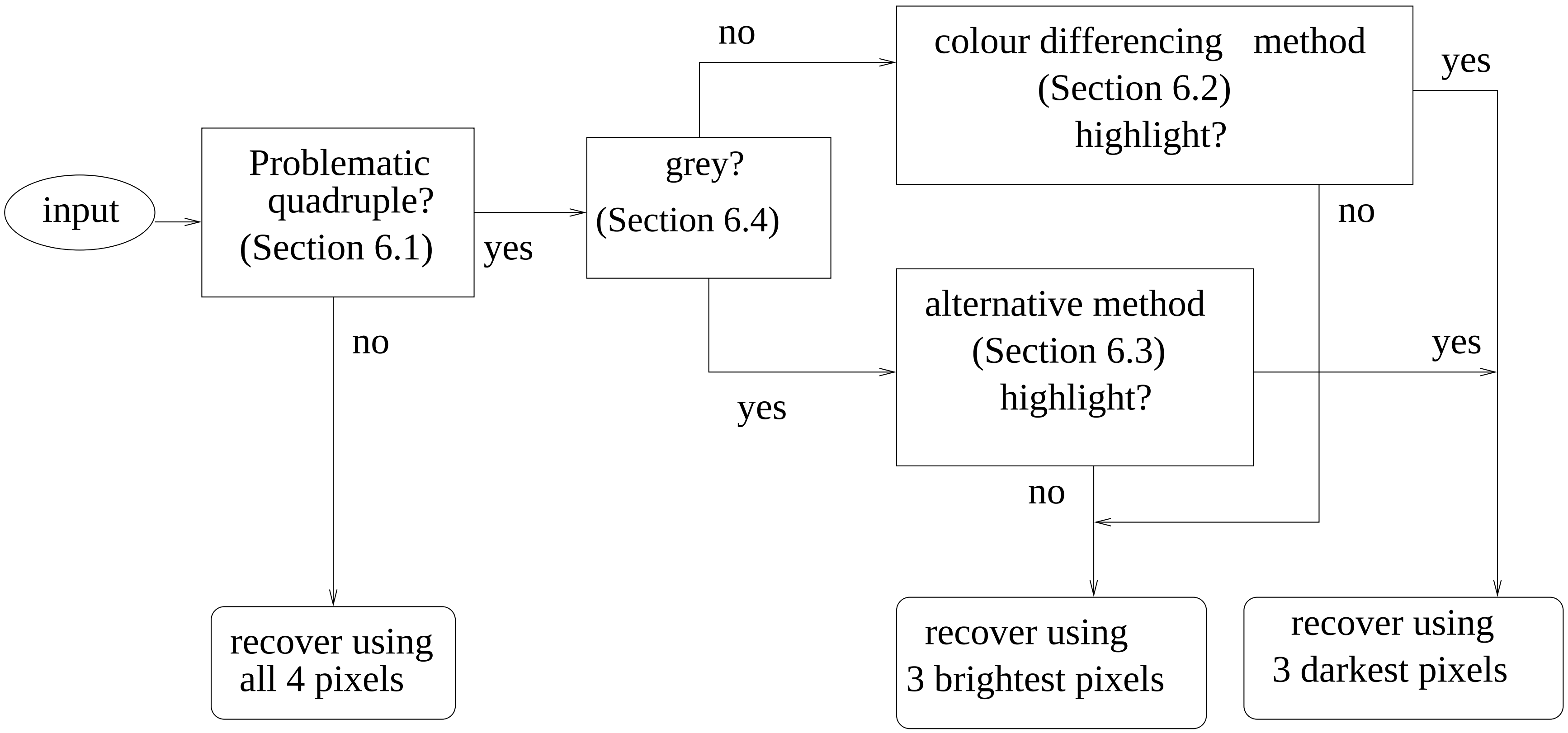
   - ***highlight*** $\Rightarrow$ $\mathbf{n}$ is close to $\mathbf{v}^b$ (spec. dir. of $\mathbf{L}^b$)

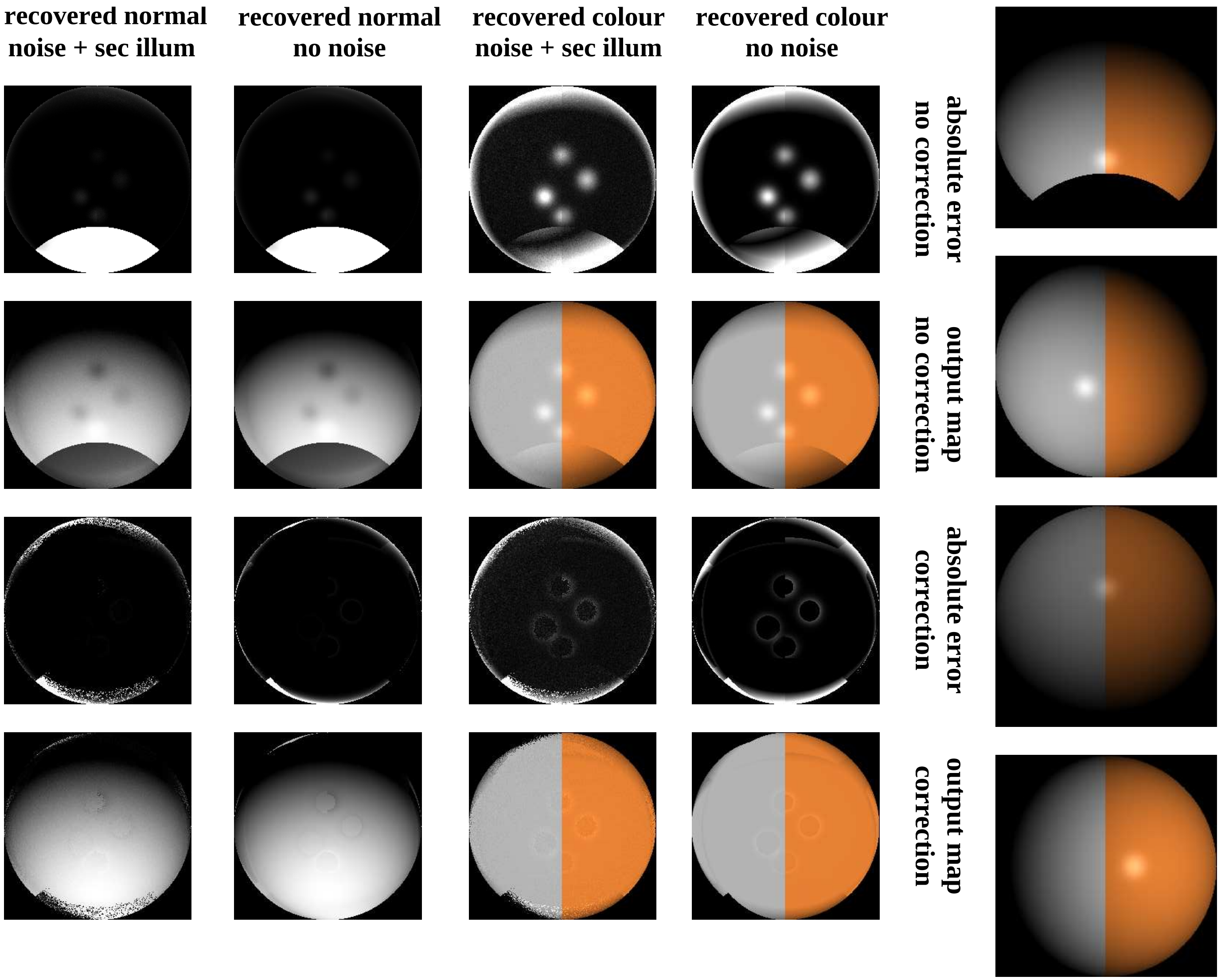
   $$d_{highlight} = 1 - \mathbf{n}^T \mathbf{v}^b$$

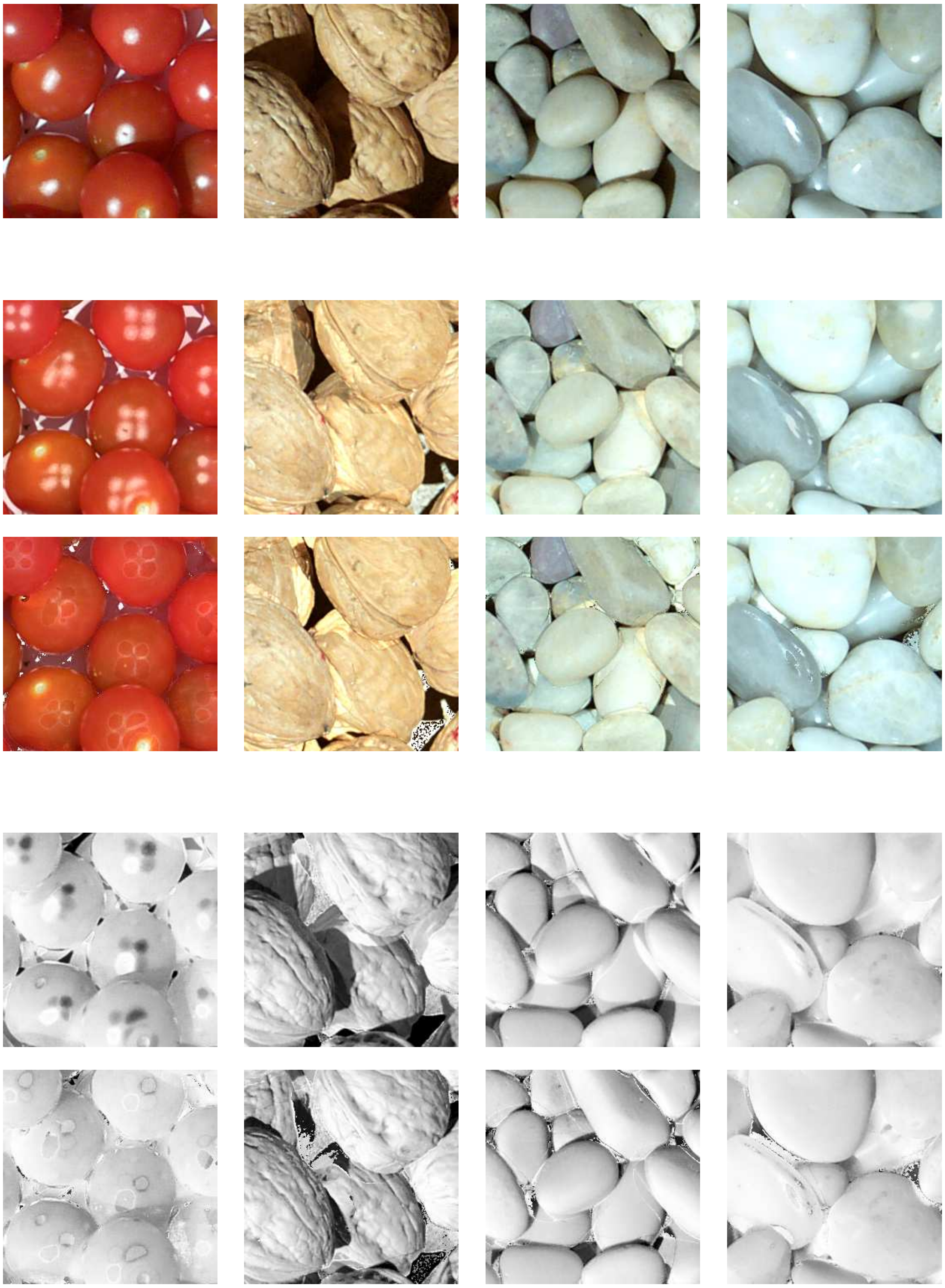
   - ***shadow*** $\Rightarrow$ $\mathbf{n}$ is close to shadowline of $\mathbf{L}^d$

   $$d_{shadow} = 1 - \mathbf{n}^T \mathbf{l}^d$$

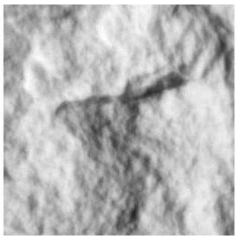
4. Choose between the two situations using a threshold

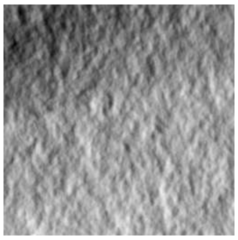
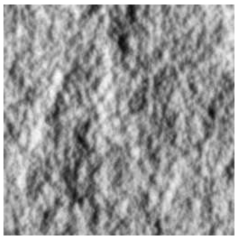
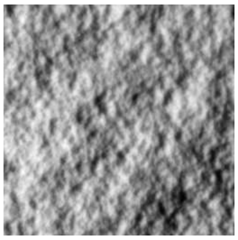
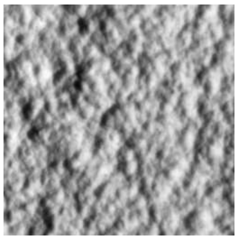
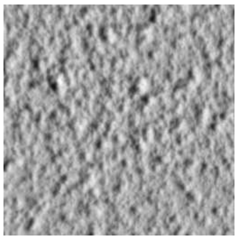
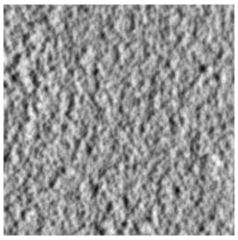
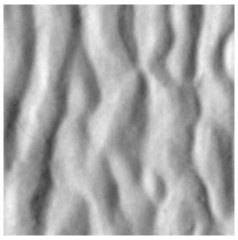
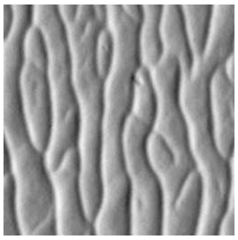
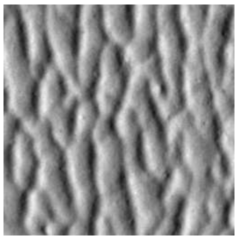
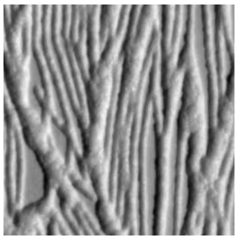
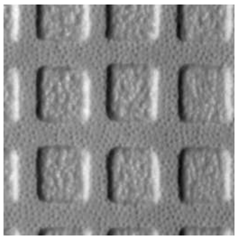
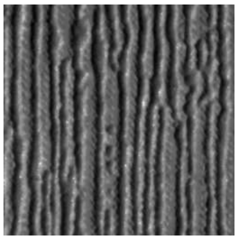

input
Problematic quadruple? (Section 6.1)
yes
no
grey? (Section 6.4)
no
yes
colour differencing method (Section 6.2) highlight?
yes
no
alternative method (Section 6.3) highlight?
yes
no
recover using all 4 pixels
recover using 3 brightest pixels
recover using 3 darkest pixels


recovered normal
noise + sec illum
recovered normal
no noise
recovered colour
noise + sec illum
recovered colour
no noise
absolute error
no correction
output map
no correction
absolute error
correction
output map
correction



3D textures and PS technique. PhoTex database


aaa

aab

aaj

aam

aan

aao

aar

aas

aba

abj

abk

acc

acd

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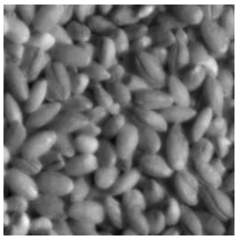
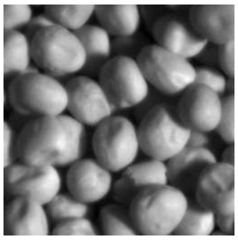
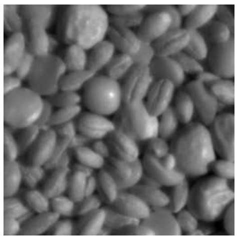
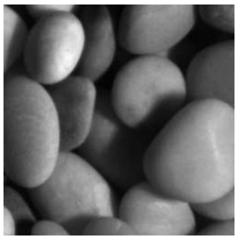
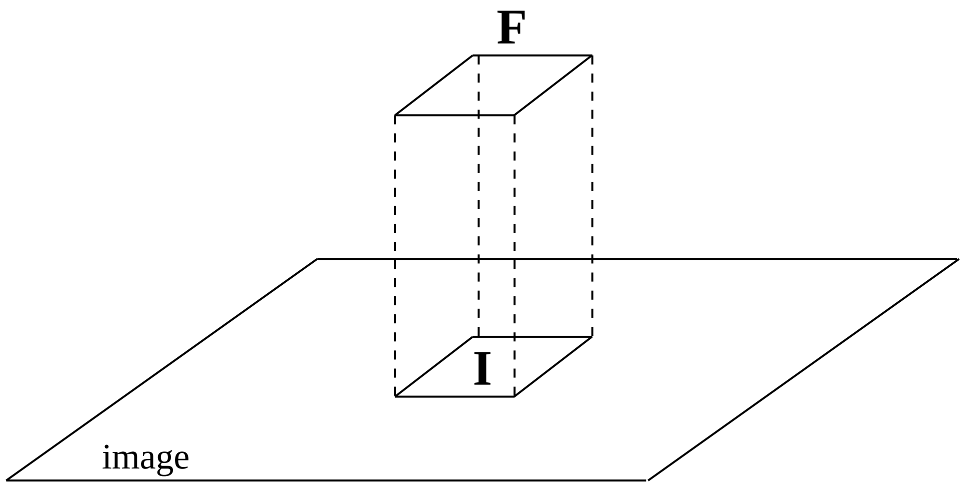

adb

adc

add

ade

adg

adh

Texture features



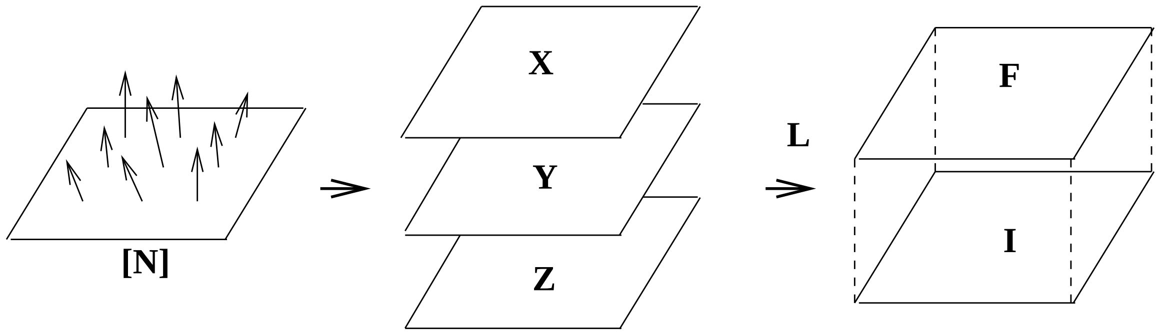
Filtering texture features:

$$r = \mathbf{F}^T \mathbf{I} \qquad \rightarrow \qquad \delta = \sigma_r^2$$

Questions:

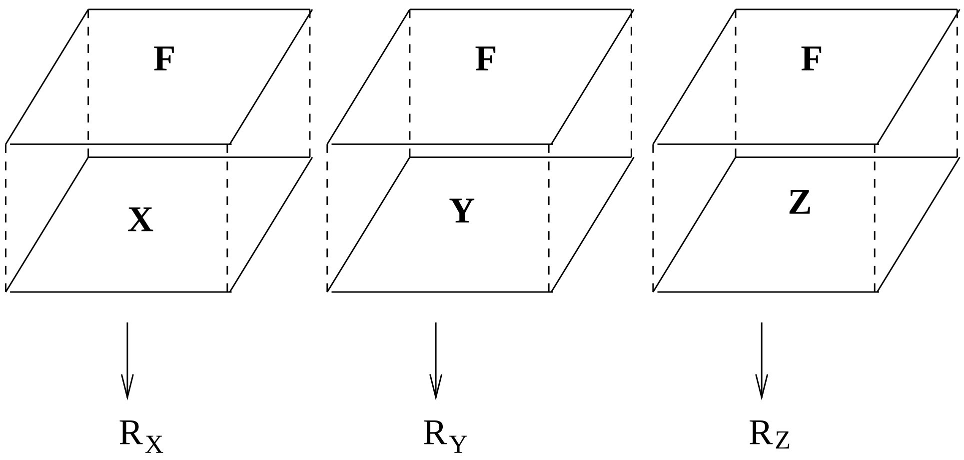
- How texture features change with changing illumination?
- How to describe a 3D texture in an illumination-independent way?

Texture features and illumination



$$r = \mathbf{F}^T\mathbf{I} = \mathbf{F}^T\left([N]\mathbf{L}\right)$$

Change order of illumination and filtering:



$$r = \mathbf{R}^T\mathbf{L} = \left(\mathbf{F}^T[N]\right)\mathbf{L}$$

We can study the behaviour of r by studying the behaviour of $\mathbf{R}$.

Texture features and joint distribution of normal components

Variance of linear combination of random variables can be expressed in terms of their covariance matrix:

$$\delta = \sigma_r^2 = \sigma_{\mathbf{R}^T\mathbf{L}}^2 = \mathbf{L}^T \Phi \mathbf{L}$$

where Φ is the covariance matrix of $\mathbf{R}$.

Elements of matrix Φ depends on cross-correlation of normal components and filter vector:

$$\phi_{ij} = \mathbf{F}^T S_{ij} \mathbf{F}$$

where matrix S_{ij} is the covariance matrix between N_i and N_j:

$$S_{ij}[kl] \equiv \mathrm{cov}\{N_k[i], N_l[j]\}$$

Matrices $S_{ij}[kl]$ are intrinsic to a 3D texture and depend on neither illumination nor filter vector.

Behaviour of texture features under changing illumination direction

$$\mathbf{L} = \mu \left(\cos\tau \cos\sigma, \sin\tau \cos\sigma, \sin\sigma \right)^T$$

where τ is tilt angle, σ is slant angle, and μ is strength of illumination.

Therefore:

$$\delta(\mu, \sigma, \tau) = \mu^2 \left[\cos^2\sigma (\phi_{11} \cos^2\tau + 2\phi_{12} \cos\tau \sin\tau + \phi_{22} \sin^2\tau) \right.$$

$$\left. + 2\cos\sigma \sin\sigma (\phi_{13} \cos\tau + \phi_{23} \sin\tau) + \phi_{33} \sin^2\sigma \right]$$

Dependence on τ:

$$\delta(\tau) = \mathcal{A}_\tau \sin(2\tau + \alpha) + \mathcal{B}_\tau \sin(\tau + \beta) + \mathcal{C}_\tau$$

Dependence on σ:

$$\delta(\sigma) = \mathcal{A}_\sigma \sin(2\tau + \gamma) + \mathcal{B}_\sigma$$

Previous model

(Heriot-Watt University, Edinburgh)

- Texture surface is represented as heights (albedo variation is excluded)
- Linearised rendering with respect to gradient $\Rightarrow$ only small variation in x and y component. Component z is almost 1.
- Calculate texture features in frequency domain as energy estimation after filtering.

Dependence on τ:

$$\delta(\tau) = \hat{\mathcal{A}}_\tau \sin(2\tau + \alpha) + \hat{\mathcal{B}}_\tau$$

Dependence on σ:

$$\delta(\sigma) = \hat{\mathcal{A}}_\sigma \cos^2 \sigma$$

This model describes feature slant response as scaling, and tilt response as sine of double tilt angle.

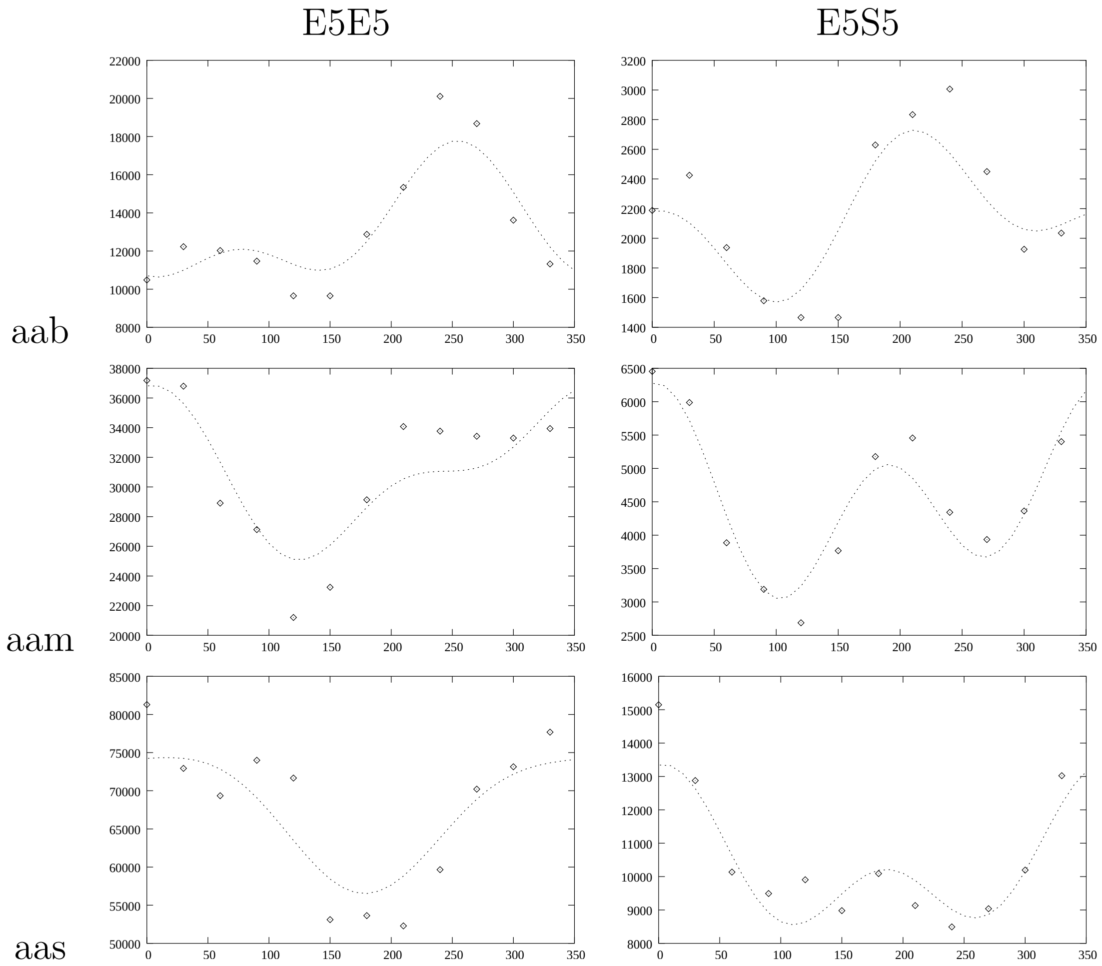
It is a partial case of our model.

We can use several different filters, and consider effects of changing illumination in the **feature space**.

Laws filters: Each filter is a matrix made up as the outer product of two of the followingfilters: E5 = (1 2 0 -2 -1), S5 = (-1 0 2 0 -1), L5 = (1 4 6 4 1).

Thus, for example, the 2-dimensional filter E5S5 has the form:

$$\begin{bmatrix} -1 & 0 & 2 & 0 & -1 \\ -2 & 0 & 4 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & -4 & 0 & 2 \\ 1 & 0 & -2 & 0 & 1 \end{bmatrix}$$



Illumination direction recovery from a single image

Consider a bank of filters. For each filter $\mathbf{F}_k$ we can find matrix Φ_k (directly from description matrices S_{ij}).

For a given image we can calculate a *texture feature vector* $\mathbf{D} = (\delta_1, \ldots, \delta_K)$:

$$\delta_k = \mathbf{L}^T \Phi_k \mathbf{L} \qquad k = 1, \ldots, K$$

Knowing $\mathbf{D}$ and all K matrices Φ_k it is possible to reconstruct illumination direction $\mathbf{L}$.

Problem is not linear!

- exhaustive search over sampled illumination space
- linearised (suboptimal) analytic solution

Beware of imaging error! It biases texture features. Solution: evaluate prediction error, find its mean and variance.

Classification from a single image

Database of A textures, bank of K filters $\mathbf{F}_k$.

For each surface we calculate K matrices Φ_k^α, $k = 1 \dots K$, $\alpha = 1 \dots A$. We also estimate distribution of the prediction error: μ_k^α and σ_k^α.

Given image, calculate its texture feature vector $\mathbf{D}$.

Calculate distance from $\mathbf{D}$ to the α-th manifold:

$$d^\alpha = \min_{\mathbf{L}} \{f^\alpha(\mathbf{L}, \mathbf{D})\}$$

where $f^\alpha(\mathbf{L}, \mathbf{D})$ is the weighted distance between $\mathbf{D}$ and feature vector, produced by $\mathbf{L}$:

$$f^\alpha(\mathbf{L}, \mathbf{D}) = \sum_{k=1}^{K} \left[\frac{\delta_k - \mathbf{L}^T \Phi_k^\alpha \mathbf{L} - \mu_k^\alpha}{\sigma_k^\alpha} \right]^2$$

Associate the image with texture β on which minimum distance is achieved:

$$d^\beta = \min_\alpha \{d^\alpha\}$$

Name	Proposed model		Sinusoidal model	
	Classification	Original surface is in the top three	Classification	Original surface is in the top three
aaa	**0.69**	**1.00**	0.58	0.86
aab	**1.00**	1.00	0.86	1.00
aaj	**0.72**	**1.00**	0.58	0.86
aam	0.81	0.92	**0.83**	**1.00**
aan	0.86	1.00	**1.00**	1.00
aao	1.00	1.00	1.00	1.00
aar	1.00	1.00	1.00	1.00
aas	0.97	1.00	**1.00**	1.00
aba	**0.72**	**0.97**	0.39	0.81
abj	**0.86**	**1.00**	0.61	0.83
abk	1.00	1.00	1.00	1.00
acc	0.50	**1.00**	**0.75**	0.92
acd	**0.92**	**1.00**	0.64	0.92
ace	**1.00**	1.00	0.89	1.00
adb	**0.83**	1.00	0.72	1.00
adc	1.00	1.00	1.00	1.00
add	0.58	1.00	**0.86**	1.00
ade	1.00	1.00	1.00	1.00
adg	**0.89**	1.00	0.86	1.00
adh	0.69	0.75	**0.75**	**0.78**

Contribution of this thesis:

1. 4-source CPS in the presence of highlights and shadows
2. Performance characterisation of the above method, its design issues (threshold values etc.)
3. Application of PS technique to illumination invariant description and recognition of 3D surfaces

Further research:

- Combine colour cross-correlation with normal component cross-correlation
- Consider *joint* behaviour of texture features
- Design robust discrimiative filters specifically for a given set of textures